SURVIVAL ANALYSIS

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Meaning by Survival Time and Survival Characteristics

- A response time distribution of a non – negative random variable representing the survival time is characterized by the following three interrelated functions:
- CDF
- Response Time Density
- Hazard Rate (HR):

- CDF:- $F(t ; x) = P\{T <= t ; x\}$
- The prob of response in a small interval (t, t+Δt)per unit timeis defined as f(t).
- $S(t)=P\{T >t)=1-F(t)$
- = Prob that response doesn't occur prior to t is known as Survival Function.
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- Response Time Density
- f(t) =lim   P(t<T<t+Δt)
-         Δt→0   t
- Hazard Rate (HR): HF or HR gives rate of change in unconditional probability of response. It gives us Instantaneous Response Rate defines as:
- h (t)= lim   P{TЄ(t, t+Δt) /T>t}
-         Δt→0   Δt
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- h (t); it is the instantaneous response rate =rate of change in conditional probability of response

- h (t)=f(t)/S(t) ,a well known result in response time analysis.

Analysis of Survival Data

(1) Kaplan Meir Product Limit (PL) Method

(2) Life Table Approach

(3) Parametric Survival Models/Hazard –based Models

(a) Additive Models

(b) GPM

(4) Semi-parametric Model Approach

Cox PH Model

(5) Dose-Response Models

Kaplan Meir Product Limit (PL) Method

- Survival Prob is calculated at each point the event/death occurs

$q_i = d_i / n_i$

$p_i = 1 - q_i$

$s_i = \prod p_i = p_i . p_2 \ldots\ldots.. p_1$

$SE(s_i) = s_i$. Sq root of $\Sigma d_i / n_i(n_i - d_i)$

Well known PL curve can be plotted between event time and survival prob. Survival curves can be compared by using Logrank test

LIFE TABLE

Life Table Analysis

- Meaning by Cohort
- Meaning by Life Table

Life History of Cohort

Utility of Life Table

a) Expectation of Life /Longevity of Life

b) Studying Survival Rates/ Comparing Mortality Pattern

c) Hazard Rates of Chronic Disease Conditions

d) Studying Default Rates

e) Fixation of Premiums of Insurance Policies

- f) Give answers of questions concerning duration of life
- g) Several uses ion Demography for calculation of NRR, GRR, Population Projections /Population Growth Models
- h) Calculation of Retirement benefits
- i) Several applications in occupational health, RCH, and social sciences (estimation of labor force, widows , orphans, pregnant and lactation mothers school going children

Components of Life Table

1) l_x = No of persons aged x

2) d_x = No of persons dying between age x and x+1 years

$$d_x = = l_x - l_{x+1}$$

3) q_x = Probability that a person of age x rears will dye before reaching age x+1 years = d_x / l_x

$$p_x = 1 - q_x = l_{X+1} / l_x$$

4) Lx = Estimated total no of person years lived in aggregate by cohort at age x

$$L_x = l_x - d_x/2$$

5) Tx = Total future life in aggregate spent by cohort at fter attaining age x.

$$= L_x + L_{x+1} + L_{X+2} + L_{X+3} + \ldots\ldots\ldots\ldots$$

$$= L_x + T_{X+1}$$

6) Expected Life at age x, $e_x = T_x / l_x$

Example

- Following Life Table gives no of patients suffering from a particular chronic disease surviving for different years since diagnosis. Find probability that the patient will die within different years after diagnosis and also no of years lived in aggregate by the cohort of 1000 patients up to 5 years since diagnosis.

Covariate-based hazard models (CBHM)

- CBHM are used in response time modeling with the following objectives:
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- Prognosis or prediction of future of patients given covariates.
- To know relative importance of covariates in prediction of response.

- One may be interested in the identification of important risk factors influencing the response pattern and to know which individual is more likely to develop a particular response

GENERAL APPROACH OF CBHM MODELLING

- To express response-time characteristic of the model in term of covariates.
- HR is the convenient tool for modeling
- Parts of CBHM:

  Model when covariate = 0

  Representation of the change induced by a non-zero covariate

Use of HF in Survival Modeling

- Hazard Rate provide another important approach of modeling association between time to occurrence of a response and concomitant variables.
- HF can assumed in terms of two factors
- -Internal (ageing factor)
- -External (interaction with the environment).
- External Factor may be multiplicative are additive in nature.

TYPES OF CBHM

Multiplicative

a) **ALM**
b) **PH Model**

- **Non-multiplicative / Additive**
- **The General Product Model (GPM)**

- Hartley and Sielken (1977) proposed the GPM.

- GPM is a PH model with a polynomial dose-response factor and with a parametric restriction on the time dependent function.

- **GPM is of the Form:-**
- $\lambda\ (t, x\ ;\ \alpha\ ,\ \beta) = u\ (t\ ;\ \alpha).\ v\ (x\ ;\ \beta)\ (\alpha\ ,\ \beta > 0)$
- In case $v\ (x\ ;\ \beta)\ (\alpha\ ,\ \beta) = 1$ at $x = 0$, Then $u\ (t\ ;\ \alpha)$ = Background Rate.
- Assuming specific parametric form for $u(\ t\ ;\alpha_{\sim})$ and $v(x\ ;\beta)$, a number of covariate-based hazard models can be undertaken.

The Cox Model

The regression model proposed by Cox (1972) (cf, Kalbfleisch and Prentice,1980 for summary) presumes the hazard rate far individuals at age t and dose x to of the form

$$\lambda(t; x) = \lambda_0(t) \exp(x\beta),$$

where $\lambda_0(t) \geq 0$ is a completely unspecified function of age, $x = (x_{1,} x_2, \ldots, x_p)$ is a regression vector with component that are specified function of dose [e.g. $x=(x^1, x^2{}_{,\ldots,} x^p)$],

and $\beta=(\beta_1, \beta_2, \ldots, \beta_p)$ is a regression coefficients to be estimated from the dose- response data.

The Additive Model

- $h\ (t\ ;\ x) = \beta_0 x_{r0} + \beta_1 x_1 \ldots\ldots\ldots + \beta_p x_p$,

  β_i , i=1,2, ……, $p \geq 0$.
- Greenberg et al (1974)
- Byar et. al (1974)
- Bhattacharya et. al)1995)
- Kumar D. (2010)

THANK YOU