TESTS OF SIGNIFICANCE

DR DINESH WALIA
DEPT OF COMMUNITY MEDICINE
GMCH CHANDIGARH

- *" Statistics may be defined as "a body of methods for making wise decisions in the face of uncertainty." ~W.A. Wallis.*

Meaning of Tests of Significance:

- Statistical procedures to draw inferences from samples about population

Why required?

- Whether difference between sample estimate and population values is significant or not?
- Differences between different sample estimates significant or not?

Meaning by Significant and Non Significant Differences

Terminology

- Hypothesis
- Null Hypothesis
- Normal Curve
- Alternate Hypothesis
- Acceptance & Rejection Regions
- Right tail & left tail tests.
- Level of significance
- Type I error (α-error)
- Type II error (β-error)
- Power of the Test

Steps in Tests of Significance

- State clearly Null Hypo (Ho)
- Choose Level of Significance (α)
- Decide test of Significance
- Calculate value of test statistic
- Obtain P-Value and Conclude Ho

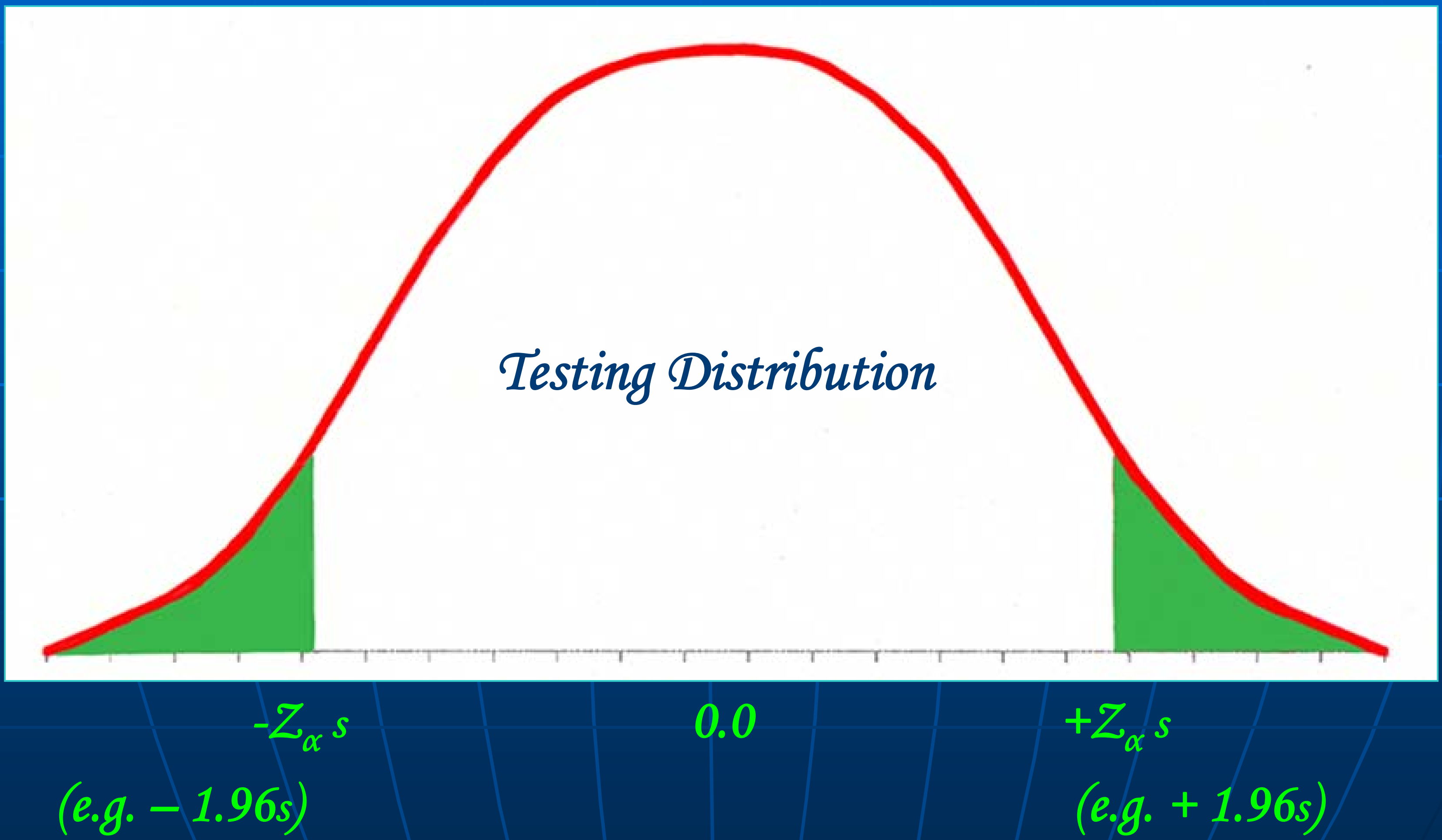
- <u>More About Acceptance and Rejection Regions</u>

Acceptance Region :Region in which Ho is accepted

Rejection Region:Region in which Ho is rejected.

- If z cal > Zα
- Then P < α → Ho is rejected
- If Zcal/ < Zα
- Then P > α → Ho is accepted
- Usually α = 0.05, May be 0.01.,0.....

Pictorial Representation of a Statistical Test



Criterion For Testing Based on Normal Distribution

- If $|z\ cal| < 1.96 \rightarrow P > 0.05$ (Non Sig)
- If $|z\ cal| > 1.96 \rightarrow P < 0.05$ (Sig)
- If $z > 2.58 \rightarrow P < 0.01$ (Highly Sig)
- If $z > 3 \rightarrow P < 0.001$ (Highly Sig)

Parametric Statistics

Z-test (for large samples) for testing significance of

- single population mean,
- difference of two population means,
- single population proportion,
- difference of two population proportions.

t-test (for small samples)

- **for testing significance of**
- single population mean,
- difference of two population means,
- paired data (Effectiveness of Drug)
- correlation coefficient.

F-Test:

- Testing equality of several means
- Testing equality of two population variances.
- One-way, Two-way and Multi-way ANOVA

Chi-square:

- Testing Associations
- Goodness of Fit
- Significance of Risk

H0: RR=1 or OR =1

Normal Test (z-Test):(For large samples, Quantitative & Qualitative Data))

(a) Test for Significance of Difference Between Two Group Means

- Ho : $\mu_1 = \mu_2$

- $$z = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- Ex. Normal Test of Means
- Mid Arm circumference (MAC) of 625 American and 625 Indian Children.

MAC	American Children	Indian Children
Mean=		15.5
SD =	5.0	5.4

- $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

20.5

- $= \dfrac{20.5 - 15.5}{\sqrt{\dfrac{5^2}{625} + \dfrac{(5.4)^2}{625}}} = 16.98$

Z cal >3 → P < 0.001 Highly Significant

- (b) Tests for two proportions
- Ho:-------------- (Qualitative Data)
- $$z = \frac{p_1 - p_2}{\sqrt{p\,q\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$
- p_1 = prop. In I Ip.} p= pooled
- p_2 = prop in II Ip.} q= prop. 1-p

- Prob. From a popn, 40 females using oral contraceptives and 60 females using other contra, were randomly selected and the number of hypertensive cases from both groups were recorded:

Type of Contra	Total Women	Hyp.
oral	40	8
others	60	15

Test the hypothesis that the prop. Of patients with hyp. Is same for the two groups.

- Ex : Normal Test of Proportions

	Gp I	Gp II
Sample size	$n_1=150$	$n_2=160$
Cp Abn	129	110

- Ho: Prop. Of CP. Abn. Is same
- $p_1 = \frac{129}{150} = 0.86$

$p_2 = \frac{110}{160} = 0.68$

- p= overall prop. = $\frac{(129+110)}{(150+160)}$

$$= \frac{(n_1 p_1 + n_2 p_2)}{(n_1+n_2)}$$

$= 0.77$

$q = 1\text{-}p = 0.23$

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 3.61$$

$P < 0.001$

(Highly Sig)

- t-test (for small samples Quantative Data)
- (a) Comparison of means of two independent samples

<u>student's t-test</u> : Ho:----------

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$\bar{x}_1$ = Mean of I group.

$\bar{x}_2$ = Mean of II group.

- n_1= no. of cases in I group.
- n_2= no. of cases in II group.
- s = pooled s.d.
- $s_2 = \frac{(n_1-1)\, s_1^2 + (n_2-1)\, s_2^2}{n_1 + n_2 - 2}$

(b) Comparison criterion:

→ 2f t cal < t

tab $(n_1 + n_2 -2)$ d.f.

Then P > 0.05 → Ho is accepted

If t cal > t tab (.05), $(n_1 + n_2 -2)$ d.f.

Then P < 0.05 → Ho is rejected

- Prob. Cumulative weight losses during insulin induced hypoglycemia for 12 patients treated with propanol
  and 12 control patients. Test the significance of difference between mean cumulative wt losses.

Group	Mean	S D
Propanol	120	10
Control placebo		8

70

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$S^2 = \frac{(n_2-1)\, s_1^2 + (n_2-1)\, s_2^2}{n_1 + n_2 - 2}$$

$$n_1 = 12\,, \qquad n_2 = 12$$

(b) Test for effectiveness of a particular drug.

(case of 2- related/dependant samples)

$$t = \frac{\bar{d}}{s/\sqrt{n}}$$

$d = AT - BT$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{\text{Sum of all diff}}{\text{No. of paired observations}}$$

s^2 = variance of differences

(s= s.d.)

$$s^2 = \frac{1}{(n-1)} \left[\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right]$$

- <u>Conclusions</u>

If $t_{cal} < t_{tab}$ 5%, (n-1)d.f.

Then $P > 0.05$, Ho is accepted.

If $t_{cal} > t_{tab}$ 5%, (n-1) d.f.

Then $P < 0.05$, Ho is rejected

- <u>Ex. Paired t-test</u>

Sr.No. Serum Cholesterol

Sr.No.	Before (x_1)	After (x_2)	d=AT-BT	x_2-x_1	$d^2 = (x_2-x_1)^2$
1	201	200			
2	231	236		5	
3	221	216		-5	
4	260	223		-27	
5	228	224	-1	-4	
6	237	216		-21	
7	326	296		-30	
8	235	195		-40	
9	240	207			

10	267	247	-20
11	284	210	-74
12	201	209	8

n=12 $\Sigma d = -242$

$\Sigma d^2 = 10766$

$$\bar{d} = \frac{\Sigma d}{n} = -20.17$$

$$S^2 d = \frac{1}{(n-1)} \left[\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right]$$

$$= \frac{1}{11} \left[10766 - \frac{(-242)^2}{12}\right]$$

Sd = 6.67

t = 3.11

tab, II df

1% level = 2.20

tab, II df

5% level

$$t = \frac{đ}{s/\sqrt{n}} = 3.02 < t \text{ tab}$$

- x^2- <u>Chi Square Test</u>

(For Qualitative Data)

Ho: There is no significant association between two attributes.

Ho: Ro=1 or Ho: R R=1

(Case control) (cohort)

<u>Formula:</u>

$$x^2 = \Sigma \frac{(O-E)^2}{E}$$

O = observed freq.

E = Expected freq.

$$E = \frac{R * C}{N}$$

	D^+	D^-	
E^+	a	b	R_1
E^-	c	d	R_2
	C_1	C_2	

- d.f. = (r-1) (c-1)

  (2-1)*(2-1)

  1 * 1 = 1

<u>NOTE :</u>

(1) Not necessarily Disease & exposure may be any two attributes A & B

(2) Not necessarily 2*2 table any number of categories of A & B.

- Short cut Method :

$$X^2 = \frac{N\,(ad-bc)^2}{R_1\,R_1\,C_1\,C_2}$$

If X^2 cal $< X^2$ tab, 5%

Then $P > 0.05 \rightarrow$ Ho is accepted

If X^2 cal $> X^2$

Then $P < 0.05, \rightarrow$ Ho is rejected

For 2 * 2 table (1 d.f.)

X^2 tab, 5% = 3.84

- Examples
- Prob 1. Test whether there is any association between death from C.H.D and ECG Abnormalities:

ECG Abn	Death due to CHD		
	Yes	No	Total
+	10	40	50
-	15	135	150
Total	25	175	200

- Ho : --------------

$$X^2 = \frac{N\,(ad-bc)^2}{R_1\ R_2\ C_1\ C_2}$$

$$= \frac{200(10 * 135 - 15 * 40)^2}{50 * 150 * 25 * 175} = \ldots$$

IInd Method: $^2 = \Sigma \frac{(0-E)^2}{E}$

E (10) = ..   X

E (40) = ..

E (15) = ..

E (135)=..

- Prob 2 :

Test whether there is any asso between smoking and hypertension?

or

Test whether there is any evidence that smoking is a risk factor for hypertension.

Smoking	HTN +	HTN -
Yes	120	280
No	30	570

Parametric Statistics: where to apply ?

- The given observations are independent
- **Observations come from normal distribution**
- The hypothesis involve population parameters
- Parametric procedures may be applied on measurement data only
- Computations are sometimes difficult
- Parametric procedures are more robust than Non-Parametric procedures.

Choice of statistical test for independent observations

		Outcome variable					
		Nominal	**Categorical**	**Ordinal**	**Quantitative Discrete**	**Quantitative Non-Normal**	**Quantitative Normal**
Input Variable	Nominal	χ^2 or Fisher's	χ^2	χ^2 or Mann-Whitney	Mann-Whitney	Mann-Whitney or log-rank (a	Student's t test
	Categorical (>2 categories)	χ^2	χ^2	Kruskal-Wallis (b)	Kruskal-Wallis (b)	Kruskal-Wallis (b)	ANOVA (c)
	Ordinal (Orde red categories)	χ^2 or Mann-Whitney	(e)	Spearman rank	Spearman rank	Spearman rank	Spearman rank or linear regression(d)
	Quantitative Discrete	Logistic regression	(e)	(e)	Spearman rank	Spearman rank	Spearman rank or linear regression (d)
	Quantitative non-Normal	Logistic regression	(e)	(e)	(e)	Plot data and Pearson or Spearman rank	Plot data and Pearson or Spearman rank and linear regression
	Quantitative Normal	Logistic regression	(e)	(e)	(e)	Linear regression (d)	Pearson and linear regression

Non-Parametric Statistics: where to apply ?

- The given observations are independent
- **Observations may not be from normal distribution**
- Based on minimum assumptions
- The hypothesis is not concerned about population parameters
- Non-Parametric procedures may be applied on weak measurement scale i.e. on count data or rank data
- Computations are simple.

Non-parametric Statistics

- Sign test: Testing median
- Wilcoxon signed rank test: Testing median
- Run test: For randomness
- Median test: For testing equality of two medians
- Mann-Whitney test: For testing equality of two medians
- Chi-square test: Testing “goodness of fit”, testing independence, homogeneity
- Kruskal Wallis: One way ANOVA
- Friedman: Two-way ANOVA


Thank You